Quantitative Analysis of Coordinated Effects

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Abstract

Mergers can affect the extent, probability, and payoffs of coordinated interaction among firms in an industry. Current analyses of coordinated effects typically provide little quantification of these effects and instead typically rely on arguments based on the number of firms, Herfindahl Index, ability to detect and punish deviations, ease of entry, and maverick firms. We offer an approach for quantifying the magnitude of the potential post-merger gains from incremental explicit collusion by subsets of firms in the post-merger industry. If the incremental payoffs to post-merger collusion are small (large), then coordinated effects are less (more) of a concern. Our approach also allows one to identify which post-merger cartels create the greatest concern and to quantify the effects of post-merger collusion on consumer surplus. The approach can incorporate divestitures and the evaluation of entry, should it occur, as well as quality improvements and cost savings resulting from the merger. We illustrate the implementation and value of this approach with applications to *Arch Coal* and *Hospital Corporation*.

*The views expressed are those of the authors and do not necessarily reflect the views of the Federal Trade Commission or its individual Commissioners.

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1 Introduction

Mergers tend to create incremental opportunities for coordinated behavior. This is recognized as a concern in Section 2 of the Horizontal Merger Guidelines (HMG or Guidelines) of the Federal Trade Commission (FTC) and Department of Justice (DoJ).[1] The Guidelines point to a need to understand the incremental incentives for, and payoffs from, coordinated behavior as a consequence of a merger. They also point to a need to understand the incremental expected deadweight loss created from a merger as a result of increased opportunities for coordinated behavior, and perhaps more relevant from the perspective of social policy, the extent to which a merger diminishes consumer surplus through the increased opportunities for coordinated interaction.

In an industry, at any time, any given subset of firms may be involved in a particular degree of coordinated interaction, ranging from static non-cooperative behavior to, at the other extreme, explicit collusion, where the subset of firms essentially function as one corporate entity.[2] The coordinated interaction between any subset of firms may depend on the coordinated interaction of other subsets of firms, where these subsets of firms may overlap. The probability of a particular configuration of coordinated interactions will depend on features of the firms, industry, and market, some of which will be observable and others that will not be. Finally, there will be payoffs for each firm associated with any given configuration of coordinated interactions among firms in an industry. With these three components—the configurations of coordinated interaction, probability of each configuration, and the payoff to each firm and the industry from each configuration—conceptually, expected payoffs can be calculated for each firm and the industry. Ideally, one would do these calculations for both the pre-merger and post-merger market, where the contrast would provide the impact of the merger with regard to coordinated effects.

Current merger analysis of coordinated effects tends to focus on questions such as: Will the merger cause the Herfindahl index to rise substantially? Will the merger absorb a "maverick" firm or otherwise negatively affect a "maverick" firm? Will the merger allow conspirators to detect deviations by other conspirators more easily? Will punishment of deviators be easier or more effective?

[1] http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html.

[2] This latter polar extreme is equivalent to a merger among the subset of firms.

Although the Herfindahl index is easy to calculate, the change in the Herfindahl from pre-merger to post-merger is merely suggestive of potential incremental coordinated effects issues. Since there is no accepted definition, empirical or otherwise, for a "maverick" firm,[3] the second question is largely ambiguous. The last two questions, although rooted in the Folk Theorem and the repeated game literature, result in "dinner party" stories,[4] where qualitative conclusions such as "fewer firms make coordinated interaction more likely" are the norm.

Coordinated effects analysis could benefit from further development of a systematic framework that provides quantifiable content and foundations for predicting post-merger conduct. In this paper, we begin from a premise that firms respond to incentives. Payoffs drive behavior. If the payoff from taking the action is small, firms are unlikely to incur costs to seek a way to undertake the action. However, if the payoff to an action is large, firms are likely to incur costs to seek a way to undertake the action. If the payoff to a given coordinated interaction is large, then firms will have an incentive to seek ways to achieve it. From this perspective, quantifying payoffs to possible configurations of coordinated interaction is important.

However, the extent of coordinated interaction can have a large range even among a given subset of firms. The firms may be highly competitive, or they may recognize their mutual interdependence but take no steps beyond the recognition, or they may take actions that have the intent to signal to one another some aspect of coordination, or they may engage in explicit discussions to suppress rivalry. The infinite number of possibilities implies a potentially infinite number of payoff calculations.

We propose a relatively simple set of calculations be conducted as a regular part of any merger analysis, namely the calculation of the post-merger payoffs to fully explicit collusion by all potential subsets of the remaining firms in the industry. Because some lesser kind of coordinated interaction is possible, the proposed analysis produces a bound on the effect of coordination. These are a relatively simple set of calculations because much of the groundwork for doing them has already been laid through unilateral effects analysis.

[3] For example, it is unclear how one would formulate a statistical test for the null hypothesis that a given firm was a "maverick." One aspect of a "maverick" is clear – if not part of the merger, their participation in post-merger coordinated interaction will be relatively low. It is important to note that the explicit mention of mavericks in the HMG implies an explicit recognition that all-inclusive explicit collusion is far from the leading concern regarding post-merger coordinated interaction.

[4] See Baker (2002).

Standard unilateral effects analyses with regard to mergers investigate, in a static context, the impact of the proposed decrease in industry size on interfirm interaction. As stated in the Guidelines,[5]

> A merger may diminish competition even if it does not lead to increased likelihood of successful coordinated interaction, because merging firms may find it profitable to alter their behavior unilaterally following the acquisition by elevating price and suppressing output.

At first glance, it might appear that unilateral effects analyses would be distinct from coordinated effects analyses by construction. An analysis conducted in a static context, controlling for the likelihood of successful coordination, might seem unsuited to address questions of coordination. However, unilateral effect analyses investigate the impact on pricing of the reduction in the number of market participants from n to $n-1$.[6] This is the nature of a merger. A merger constitutes explicit collusion between two firms, where the terms of the collusion are contractible. In this light, unilateral effects analyses can be viewed as addressing the impact on pricing when two firms, who were acting as non-cooperative rivals, engage in contractually-binding explicit collusion. In other words, standard unilateral effects analyses are an investigation of a polar extreme of coordinated effects. Nothing prevents these analyses from being extended in a number of directions. A unilateral effects analysis that investigates a change from n to $n-1$ can be extended to investigate a change from $n-1$ to $n-2$. Furthermore, the analysis can address each of the possible ways of going from $n-1$ to $n-2$. In general, the analysis can be extended to look at a change from $n-1$ to $n-k$ where $2 \leq k \leq n-1$.[7]

We propose a three-step process. First, select an appropriate model of competition. This might be quantity competition, differentiated products price competition, bidder competition within an auction of procurement, or some other model of competition that incorporates the salient features of a given industry. Second, fit and/or calibrate the model to the pre-merger market and relevant features of the pre-merger firms, such as their market shares. Third, within the fitted and/or cal-

[5] HMG at Section 2.2.

[6] There can be exceptions. In *Arch Coal* the proposed merger was coupled with a proposed divestiture, thus the proposed merger left the number of firms in the industry unchanged.

[7] Of course, $k = (n-1)$ is the all-inclusive cartel.

ibrated competitive framework, calculate the effect of the merger and the effects of various post-merger explicit collusion scenarios.

Our approach does not displace any existing analysis. Rather, it is an incremental augmentation to existing analyses. Nevertheless, the incremental gain to merger analysis from this approach to quantifying coordinated effects is potentially large. The analysis can be used to quantify the payoff to all market participants from incremental explicit collusion between any pair, or any subset, of remaining firms in the industry. These payoff calculations may reveal that incremental coordinated interaction is a significant concern, or they may reveal that there is little concern, or they may reveal that incremental coordinated interaction is a significant concern between a specific subset of firms.

Although the proposed analysis does not offer a direct implication for the probability of a specific configuration of coordinated interactions, no current analysis provides any direct quantifiable insight in this regard. However, by quantifying the incremental payoff to any subsequent collusion, and assuming the probability of such collusion is increasing in the incremental payoff, our analysis can augment existing analyses by offering indirect qualitative probability assessments.

The paper proceeds as follows. Section 2 sets the proposed analysis within the Guidelines. Section 3 describes how this proposed analysis could have been applied in two past merger cases. Section 4 concludes.

2 The Proposed Analysis and the Guidelines

The HMG treatment of coordinated effects focuses on the capacity of a merger to increase coordination by firms that remain in the relevant market with respect to price, quality, or other dimensions of competition. Section 2.0 of the HMG observes that "[c]oordinated interaction is comprised of actions by a group of firms that are profitable for each of them only as a result of the accommodating reactions of the others."[8] Successful coordination requires "reaching terms of coordination that are profitable to the firms involved."[9]

The Guidelines' analysis of possible future coordination, and the increased profitability it may generate, focuses chiefly on the presence or absence of industry con-

[8] HMG, at Section 2.1.

[9] Id.

ditions that would facilitate the completion of the three tasks – the formulation of a consensus, the detection of deviations from the consensus, and the punishment of cheaters – that are necessary to successful coordination.[10] To this end, the U.S. antitrust agencies "not only assess whether the market conditions for viable coordination are present, but also ascertain specifically whether and how the merger would affect market conditions to make successful coordination after the merger significantly more likely."[11] The assessment of post-merger performance outcomes "includes an assessment of whether a merger is likely to foster a set of common incentives among remaining rivals, as well as to foster their ability to coordinate successfully on price, output, or other dimensions of competition."[12]

Like the HMG, our analysis is concerned with the incentives of firms in the relevant market, but with a somewhat greater emphasis. Our approach focuses greater attention on how a proposed merger affects the perceptions of the industry participants of their post-merger profitability and how perceptions of greater or lesser profitability affect their incentives to strive to solve the tasks (consensus building, detection, and punishment) that must be accomplished for coordination to succeed. Our approach assumes that firms will try harder to solve the coordination tasks as the perceived positive impact on profitability increases.

3 Analyses with Applications to Past Cases

Two significant coordinated effects cases are *Arch Coal* and *Hospital Corporation*. We illustrate our approach to quantifying coordinated effects within the context of these two cases. In both cases, our approach involves extending unilateral effects analysis to consider the effects of hypothetical mergers beyond those proposed; however, we base the quantification of those effects on an auction model for the *Arch Coal* case and on a model of differentiated products price competition for the *Hospital Corporation* case. Other models that allow this type of quantification could be used in other cases as appropriate.

In Section 3.1, we provide background for the *Arch Coal* case and then illustrate how the characteristics of the relevant market can be calibrated to an auction model.

[10] See Federal Trade Commission & U.S. Department of Justice, Commentary on the Merger Guidelines 18-25 (2006).

[11] Id. at 18.

[12] Id.

That model can then be used to quantify the effects of coordination by various subsets of firms in the market. In Section 3.2, we take a similar approach for the *Hospital Corporation* case. First, we provide some background, then we show how one can calibrate a differentiated products price competition model to the market, and finally we show how one can use the model to quantify the impact of coordinated effects.

3.1 Quantifying Coordinated Effects Using a Model of Auctions and Procurements with an Application to *Arch Coal*

We begin with some background on *Arch Coal* in Section 3.1.1, and then in Section 3.1.2, we describe how our approach can be implemented using a model of auctions and procurements. In Section 3.1.3, we discuss implications for *Arch Coal*.

3.1.1 Background on *Arch Coal*

Electric power utilities burn coal to generate electricity. Coal from the South Powder River Basin (SPRB) in northeastern Wyoming has low sulphur content (advantageous for environmental compliance) and high heat content. Prior to 2004, five major firms mined coal in the SPRB: Arch Coal, Peabody, Kennecott, Triton, and RAG. In 2004, one of these firms, Arch Coal, proposed the purchase of a competitor, Triton, where one of Triton's mines would be immediately divested to Kiewit, leaving five firms in the industry, albeit with a different industry concentration than before the proposed merger.

The FTC opposed the merger largely on the grounds that coordinated conduct would increase after the merger. A primary argument of the FTC was that future supply restrictions were likely because the gains to coordinated behavior would increase as a consequence of the merger.[13] However, the District Court argued that the competitive bidding procedures used by utilities to acquire coal from the SPRB producers would frustrate coordination.

In what follows, we examine coordination within the context of a competitive bidding process, absent any supply restricting behavior. The competitive bidding analysis allows us to quantify the effects of post-merger potential explicit collusion

[13] Arch Coal, 329 F. Supp 2d 109, 2004-2 Trade Cases P74,513, p.21.

by the bidders. This analysis is appropriate with homogeneous industrial products. Differentiated consumer products will be analyzed differently in Section 3.2.

3.1.2 A Model of Collusion at Auctions and Procurements

Explicit collusion by bidders at auctions and procurements has received attention in the economics literature over the past two decades.[14] Analytically, explicit collusion at an auction/procurement has been treated as if the bidders became one bidding entity. Attempts have been made to contrast the susceptibility of different auction/procurement schemes to collusion. Specifically, contrasts have been made between the oral ascending bid auction (or second price sealed bid auction) and the first price sealed bid auction.[15]

An oral ascending bid auction is thought to be more susceptible to collusion than a first price sealed bid auction. At an oral ascending bid auction, the highest-valuing member of the cartel or ring need not change their behavior from what they did acting non-cooperatively. The collusive gain is secured by having all other ring members not bid at the auction. But, the behavior of the highest-valuing member insures that no ring member can profitably defect on the agreement. In contrast, to secure a collusive gain at a first price auction, the highest valuing bidder must change their bidding behavior from what they would have done acting non-cooperatively. This creates an opportunity for cheating by the other ring members who are suppressing their bids, but wondering if they could profitably outbid the suppressed bid of the ring member with highest value.

To see why this is relevant for merger analysis, note that collusion at an oral ascending bid auction does not change the expected payoffs, or ex post payoffs, for any non-colluding bidder, and it does not affect their bidding behavior. However, collusion at the first price auction does impact the expected payoff for non-colluding bidders, and it does affect their bidding behavior. When considering the merger of

[14] Multiple object auctions and procurements have received less attention than single object auctions and procurements, and independent private value models have received more attention than affiliated and common value models. The emphases reflect both the perceived relevance of various models as well as their analytic tractability.

[15] See, e.g., Robinson (1985) and Marshall and Marx (2006). In a first price auction, sealed bids are submitted and opened simultaneously. The winner is the one who submitted the highest bid, and they pay the amount of their bid. For a second price auction, sealed bids are submitted and the highest bidder also wins, but they pay the amount of the second highest bid. An oral ascending bid auction, also known as an English auction, is strategically equivalent to a second price auction—bidders submit ever increasing bids until only one bidder remains active.

two firms in an industry, we believe it is unreasonable to think that the appropriate tool for analyzing coordinated effects would leave the expected payoff and ex post payoff of non-merged firms completely unchanged from the merger. Rather, it seems far more reasonable that the merger would produce a benefit beyond just that for the merged firms; it would also produce a benefit for the non-merged firms. The suppression of competition between the merged entities is typically not a benefit that can be captured exclusively by the merging firms. Some of the suppression of rivalry will benefit non-merging firm as well.[16] Thus, it seems most reasonable to work within the context of a first price auction when analyzing certain industries.[17]

Unfortunately, first price auctions are not trivial to analyze. The differential equations and boundary conditions that define the unique Nash equilibrium are almost always analytically intractable. Numerical methods are required to solve them. But, under somewhat mild conditions, the solution is unique. This is a positive attribute when considering the use of the framework for policy analysis since we avoid the ambiguities created by multiple equilibria as we move from one industry configuration (n firms) to another ($n - 1$ firms).

A recent development in the analysis of asymmetric first price auctions removes a big constraint in the use of this analysis for quantifying coordinated effects. Gayle and Richard (2005) have defined a topology for the relevant Taylor series expansions, and have provided corresponding numerical methods, so that any underlying distribution of values or costs, even empirical ones, can be accommodated. Prior to this work, one was constrained to work with power functions and extreme value distributions, neither of which may have adequate flexibility to account for the richness of a given merger environment.

Although an analysis based on a single object auction/procurement analysis, by definition, will never entail a reduction in quantity brought to market, our proposed analysis provides a bound to the payoffs that collusion will produce. Fully explicit collusion without a reduction in quantity sold, or purchased, as a consequence of the collusion is an upper bound on the potential harm from incremental collusion.

We begin with a simple example using power distributions so that the underlying

[16] Duso, Gugler, and Yurtoglu (2005) examine the abnormal returns of non-merging firms around the announcement of a merger and other events related to antitrust enforcement for evidence of anti-competitive effects.

[17] A first price procurement may seem more natural. However, for simple procurements there is no meaningful difference between a procurement and an auction.

methodology and lines of argument can be understood. Calculations based on the example are shown in Figure 1.

Four bidders		Three bidders		Two bidders		One bidder	
Bidder type	Expected surplus	Bidder type	Expected surplus	Bidder type	Expected surplus	Bidder type	Expected surplus
3	6.16	5	13.52	6	23.83	7	87.50
2	4.34	1	3.76	1	7.25	Total surplus	87.50
1	2.24	1	3.76	Total surplus	31.08	Expected revenue	-
1	2.24	Total surplus	21.05	Expected revenue	54.30		
Total surplus	14.99	Expected revenue	65.37				
Expected revenue	72.38						
		4	9.35	5	14.74		
		2	5.38	2	7.75		
		1	2.81	Total surplus	22.48		
		Total surplus	17.54	Expected revenue	64.43		
		Expected revenue	69.62				
		3	7.02	4	10.89		
		3	7.02	3	8.86		
		1	2.58	Total surplus	19.75		
		Total surplus	16.61	Expected revenue	67.70		
		Expected revenue	70.77				
		3	6.52				
		2	4.59				
		2	4.59				
		Total surplus	15.71				
		Expected revenue	71.74				

Figure 1: Uniform power distributions

The first column is the starting point. There are four firms in the industry, which will be treated as bidders at an auction. Each bidder has a type. The first bidder's type is "3." Think of this as meaning that this bidder gets to take three draws from a uniform distribution on zero to 100, and retain the highest of those draws as its value for the item. The bidder labeled "2" gets to take two draws. The bidders labeled "1" gets one draw. The expected surplus column provides the average payoff that the bidder can expect from participating in the auction. The total surplus is just the sum of the expected surpluses. The expected revenue is what the auctioneer can, on average, expect to receive for the item being sold.

The next major column is labeled "Three Bidders." Consider the entries in the first cell. The bidder labeled "5" gets five draws from the uniform distribution on zero to 100 and acts as if its value is the highest of those. The other two bidders only get one draw. To see how this case relates to the previous one, note that there are still two bidders labeled "1," but we have gone from two bidders labeled "3" and "2" to one bidder labeled "5". Recall that the bidder labeled "3" took three independent draws from the uniform distribution and treated its value as the highest of those. The bidder labeled "2" took two draws and treated its value as the highest of those. If those two bidders shared their value draws, then they would become a single bidder who had five independent draws and bid as if its value was the highest of those five draws. This is exactly the case described in the second major column, first cell. In other words, "5,1,1" is just a merger of "2" and "3" from the case of "3,2,1,1." The remainder of the table is read in similar fashion.

The first point to note from Figure 1 is that the comparison of the first major column to the second major column falls within the domain of standard unilateral effects analysis. The third major column is not considered in either standard unilateral effects analysis or coordinated effects analysis. However, we believe that the third major column addresses many of the queries posed regarding coordinated effects in the Guidelines. Specifically, the incremental payoffs to any form of post-merger explicit collusion can be directly quantified. The analysis is grounded in theory, and the assumptions are exposed for all to consider and probe.

The payoff changes associated with incremental collusion do not offer any explicit statement about the chance of that particular collusion occurring, but they do offer an implicit statement—it is reasonable to presume that the probability of incremental collusion is increasing in the payoff to that collusion. This may be viewed as a limitation to the analysis, but no other coordinated effects analysis is capable of producing a quantifiable probability.

As an illustration, assume the example above represents a specific industry that has four firms to start and consider what we might learn from the example regarding coordinated effects.

- *Incremental payoffs.* Consider any proposed merger (one of the four cells in column 2). Now consider one of the three cells in column 3 that may emerge as a cartel from post-merger incremental bilateral collusion. It is clear that the biggest payoff in column 3 comes from a duopoly with a highly asymmetric structure "6,1." The incremental payoff is largest in going to "6,1," as opposed to any other incremental collusion that is possible regardless of the starting point in column 2.

- *Merging firms anticipating future coordinated effects.* "3,3,1" is more likely to be approved on the grounds of unilateral effects than "5,1,1" since the impact on auctioneer expected revenue is much lower, but there is significant danger in the approval of "3,3,1" for future coordinated behavior. Specifically, there is a bigger incremental payoff to "6,1" from the starting point of "3,3,1" than from "5,1,1." In addition, when starting from "3,3,1," each of the "3" bidders is an obvious beneficiary from the collusion, whereas some type of unequal split would have to be formulated to get "5" to agree to the incremental collusion.

- *Maverick-firm.*- Suppose that in considering the bidders comprising "3,3,1" we were able to identify one of the "3" bidders as a maverick. Now the concerns regarding "6,1" from the merger producing "3,3,1" are mitigated.

The analysis could be extended in a number of directions. Tables of results regarding specific extensions can be found in the Appendix.

- *Competitive-fringe.* A competitive fringe could be introduced. In Appendix A.1, the fringe is assumed to be 4 smaller firms. Quantification of the effects of explicit collusion with the presence of a fringe is then possible. As one would expect, incremental collusion is not as profitable with a fringe as opposed to the absence of a fringe, but the techniques described here allow a researcher to specify a fringe that matches the fringe of the industry in question. The discussion can then be focused on the best calibration and implied results, rather than qualitative assertions about the impact of a fringe.

- *Divestiture- and- entry.* If a competitive fringe can be introduced with such ease, then clearly the framework can provide quantification for entry and/or divestitures (we will see the latter in the next section within the context of a calibrated *Arch-Coal*-example).

- *Efficiencies-from-the-merger.* It is common for merging firms to argue that the merger will generate efficiencies, such as cost savings or other productive efficiencies. In Appendix A.2, we present one way to capture efficiencies from mergers. In the previous example in the text, suppose it is asserted that a merger of the "2" type with one of the "1" types will result in an efficiency gain. The merged firm can be modeled as a higher type than just a "2+1." For example, it can be modeled as a "5" type. The post merger non-cooperative world would then have a "3" type, a "5" type, and a "1" type. The example captures the benefits in terms of efficiency gains from the merger. However, the example also shows that the incremental payoffs to post-merger explicit collusion between the "3" type and the merged entity are high, much higher than what they would be in the absence of efficiency gains from the merger. In fact, this example highlights a caution that many merger cases that are argued on the basis of strong efficiency gains need to be carefully examined for post-merger coordinated effects.

- *Virtually unrestricted calibration.* The researcher is largely unrestricted in the choice of distribution that they select to describe the initial status of the industry. Appendices A.3 and A.4 illustrate that different types of distributions and mixtures of different types of distributions that can be accommodated using the methods of Gayle and Richard (2005).

It is common for the focus of attention in merger cases to be on the last column of Figure 1, which shows an all-inclusive cartel. This focus is largely misplaced. Not even the International Vitamins Cartel was all-inclusive for many vitamins. The merger guidelines recognize the importance of "maverick firms," whatever they are, they are not firms waiting to join cartels. The emphasis on all-inclusive collusion may stem from the economics literature which largely emphasizes the all-inclusive cartel since in the equilibrium of simple models there are often no reasons for a cartel to be less than all-inclusive.

3.1.3 Application to *Arch Coal*

We can now turn our attention to an example that has been calibrated to match the recent *Arch Coal* merger case. *Arch Coal* is well suited to this kind of auction/procurement analysis. The product is homogeneous and most buyers use competitive procurements. The general methodology explained herein can be extended to other unilateral effects analyses as described in the following section.

The results of the calibration are shown in Figure 2.

The values are distributed over [0,1] according to beta distributions.[18] The parameters were calibrated to match reported production shares for each firm.[19]

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[18] The beta distribution has two parameters, typically denoted α and β, and has density function $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du}$.

[19] One of the advantages of the analysis is that other distributions can be used. Calibrations

Pre-merger		
Firm	Expected surplus	Prob. of Winning
Kennecott	0.046	29%
Peabody	0.046	29%
Arch	0.024	18%
Rag	0.016	13%
Triton	0.016	13%
Total	0.149	HHI=2258
Expected revenue	0.656	
Post-merger		
Firm	Expected surplus	Prob. of Winning
Kennecott	0.047	29%
Peabody	0.047	29%
Arch	0.035	24%
Rag	0.016	13%
Kiewit	0.007	6%
Total	0.151	HHI=2419
Expected revenue	0.656	

Pre-merger		
Firm	Expected surplus	Prob. of Winning
Kennecott/ Peabody/Arch	0.155	60%
Rag	0.030	20%
Triton	0.030	20%
Total	0.215	HHI=4372
Expected revenue	0.577	
Post-merger		
Firm	Expected surplus	Prob. of Winning
Kennecott/ Peabody/Arch	0.191	64%
Rag	0.038	24%
Kiewit	0.017	12%
Total	0.247	HHI=4854
Expected revenue	0.543	

Pre-merger		
Firm	Expected surplus	Prob. of Winning
Kennecott/ Peabody/ Arch/Rag	0.241	70%
Triton	0.055	30%
Total	0.296	HHI=5777
Expected revenue	0.488	
Post-merger		
Firm	Expected surplus	Prob. of Winning
Kennecott/ Peabody/ Arch/Rag	0.341	76%
Kiewit	0.047	24%
Total	0.388	HHI=6354
Expected revenue	0.384	

Figure 2: Calibration to *Arch-Coal*

To recall the history of *Arch-Coal*, Arch proposed to buy Triton but divest one of Triton's mines to Kiewit. Thus, the post-merger cells still have five firms, but Kiewit is a much smaller firm than was Triton, and Arch is bigger than in the pre-merger case.

The incremental payoff to collusion between Kennecott, Peabody, and Arch prior to the merger is 0.039, whereas after the merger it is 0.062. In other words, there is a 59% increase in the payoff to a given form of collusion after the merger than prior to the merger. We can also consider the incremental payoff to collusion between Kennecott, Peabody, Arch, and RAG prior to the merger, 0.125, versus the post-merger incremental payoff, 0.212. This is a 70% increase in the incremental payoff to a given form of collusion after the merger versus prior to the merger.

It is clear from the simulations that the FTC's concerns about coordinated effects were well grounded. As noted by the FTC, and acknowledged by the court, the change from pre-merger to post-merger absent any coordinated effects looked quite small. However, after the merger, the potential for incremental collusion, assuming it to be increasing in the incremental payoff, is substantially larger. Our analysis provides quantification for the concerns underlying the Commission's decision to prosecute.

When looking back at the arguments posed by the FTC, a more elaborate effort

can be extended to industry characteristics beyond market shares. What is and is not "best" for conducting the analysis is a legitimate question, and should be asked. This shifts the discussion away from loosely-grounded assertions and to the underlying assumptions and calibrations of a formal analysis, which is to the benefit of all involved parties.

at quantification regarding the potential for coordinated effects may have been useful to the Commission's case. The District Court, in reaching its decision, put great weight on the competitive bidding process used by utility companies to buy SPRB coal.[20] Perhaps the District Court would have attributed greater weight to a more extensive formal analysis grounded in the sealed bidding process that quantified the threat from post-merger coordinated behavior. How the Court would have ruled is not predictable, but at least the court's analysis and reasoning would have been informed more fully by issues such as the calibration of the merger's likely effects.

3.2 Quantifying Coordinated Effects Using a Model of Differentiated Products Price Competition with an Application to *Hospital Corporation*

We begin with some background on the *Hospital Corp.* case in Section 3.2.1, and then in Section 3.2.2 we describe how our approach can be implemented using a model of differentiated products price competition. Section 3.2.3 discusses some extensions.

3.2.1 Background on *Hospital Corporation*

As stated in Hospital Corporation of America v. Federal Trade Commission,[21] in 1981 and 1982, Hospital Corporation of America acquired Hospital Affiliates International, Inc. and Health Care Corporation. Before these acquisitions, Hospital Corporation had owned one hospital in Chattanooga, Tennessee, and the acquisitions gave it ownership of two more. In addition, pursuant to the terms of the acquisitions, it assumed contracts that Hospital Affiliates International had made to manage two other Chattanooga-area hospitals. So after the acquisitions, Hospital Corporation owned or managed 5 of the 11 hospitals in the area. The FTC challenged the acquisitions, saying they violated section 7 of the Clayton Act. In particular, the FTC expressed concerns about the potential for post-acquisition coordination between Hospital Corporation and the other three large hospitals in the area.

The acquisitions raised Hospital Corporation's market share in the Chattanooga area from 14 percent to 26 percent. This made it the second-largest provider of hos-

[20] Arch Coal, 329 F. Supp 2d 109, 2004-2 Trade Cases P74,513, p.30.

[21] Hospital Corporation of America v. Federal Trade Commission, 807 F.2d 1381 (December 18, 1986).

pital services in a market where the four largest firms together had a post-acquisition market share of 91 percent (as compared to 79 percent before the acquisitions).[22] The FTC concluded that the acquisitions created a danger that the largest Chattanooga hospitals would collude.

The Court decision states (at 6):

> The reduction in the number of competitors is significant in assessing the competitive vitality of the Chattanooga hospital market. The fewer competitors there are in a market, the easier it is for them to coordinate their pricing without committing detectable violations of section 1 of the Sherman Act, which forbids price fixing. This would not be very important if the four competitors eliminated by the acquisitions in this case had been insignificant, but they were not; they accounted in the aggregate for 12 percent of the sales of the market. As a result of the acquisitions the four largest firms came to control virtually the whole market, and the problem of coordination was therefore reduced to one of coordination among these four.

The decision continues (at 7):

> Moreover, both the ability of the remaining firms to expand their output should the big four reduce their own output in order to raise the market price (and, by expanding, to offset the leading firms' restriction of their own output), and the ability of outsiders to come in and build completely new hospitals, are reduced by Tennessee's certificate-of-need law. Any addition to hospital capacity must be approved by a state agency. The parties disagree over whether this law, as actually enforced, inhibits the expansion of hospital capacity. The law may indeed be laxly enforced. Not only is there little evidence that it has ever prevented a hospital in Chattanooga from making a capacity addition it wanted to make, but empirical studies of certificate of need regulation nationwide have found little effect on hospital expenditures. See Joskow, Controlling Hospital Costs: The Role of Government Regulation, ch. 7 (1981). Yet the Tennessee law

[22] These are the FTC figures as stated in Hospital Corporation of America v. Federal Trade Commission, 807 F.2d 1381 (December 18, 1986), at 4.

> might have some effect under the conditions that would obtain if the challenged acquisitions enabled collusive pricing of hospital services. Should the leading hospitals in Chattanooga collude, a natural consequence would be the creation of excess hospital capacity, for the higher prices resulting from collusion would drive some patients to shorten their hospital stays and others to postpone or reject elective surgery. If a noncolluding hospital wanted to expand its capacity so that it could serve patients driven off by the high prices charged by the colluding hospitals, the colluders would have not only a strong incentive to oppose the grant of a certificate of need but also substantial evidence with which to oppose it–the excess capacity (in the market considered as a whole) created by their own collusive efforts. At least the certificate of need law would enable them to delay any competitive sally by a noncolluding competitor. Or so the Commission could conclude (a refrain we shall now stop repeating). We add that at the very least a certificate of need law forces hospitals to give public notice, well in advance, of any plans to add capacity. The requirement of notice makes it harder for the member of a hospital cartel to "cheat" on the cartel by adding capacity in advance of other members; its attempt to cheat will be known in advance, and countermeasures taken.

To justify its prediction of probable anticompetitive effects, the FTC pointed out that: 1. demand for hospital services is highly inelastic; 2. "there is a tradition, well documented in the Commission's opinion, of cooperation between competing hospitals in Chattanooga;"[23] 3. hospitals benefit by presenting a united front in negotiations with third-party payors, particularly since hospitals are under great pressure from the federal government and insurance companies to cut costs.

3.2.2 A Model of Differentiated Products Price Competition

We present a model that allows us to quantify the benefits of coordination between HCA and the three other large Chattanooga-area hospitals, both before and after the acquisitions. This allows us to quantify the increase in incentives for coordination as a result of the acquisitions.

[23] *Hospital Corporation* at 8.

We consider a model of differentiated products price competition with 11 firms, where the products of the firms are assumed to be imperfect substitutes for one another.[24]

To calibrate the model, we refer to the Court decision for information about the market shares of the Chattanooga hospitals. There were eleven hospitals in the market. HCA's original hospital had share 14%. It acquired or took over management of four hospitals with combined share 12%. The largest hospital had share greater than 26%, and HCA's hospitals, with their combined share of 26%, together with the three other large hospitals, had combined share 91%. Consistent with this information, we craft a hypothetical with eleven hospitals that broadly captures this observed market share structure. (See Figure 3.)

Hospital	Description	Share
1	HCA	14%
2, 3, 4, and 5	HCA acquired	3%
6	largest	30%
7 and 8	large	17.5%
9, 10, and 11	small	3%

Figure 3: Target Market Shares

Then, within the context of our model, we seek a parameterization that mimics this conjectured market share structure. We assume that for all i and j, $b_i = 1$, $c_i = 0$, and $s_{ij} = 0.9$, and we choose the intercept terms a_i as follows: $a_1 = 0.887$, $a_2 = a_3 = a_4 = a_5 = a_9 = a_{10} = a_{11} = 0.874$, $a_6 = 0.898$, $a_7 = a_8 = 0.890$. Given these assumptions, the revenue shares under price competition for the eleven firms are as shown in Figure 4.

[24] As in Singh and Vives (1984), we assume a representative consumer that maximizes $U(q_1, ..., q_{11}) - \sum_{i=1}^{11} p_i q_i$, where

$$U(q_1, ..., q_{11}) = \sum_{i=1}^{11} \left(a_i q_i - \frac{1}{2} \left(b_i q_i^2 + \sum_{j \neq i} s_{ij} q_i q_j \right) \right).$$

This utility function gives rise to a linear demand structure with inverse demands given by, for $i = 1, \ldots, 11$, $p_i = a_i - b_i q_i - \sum_{j \neq i} s_{ij} q_j$. In this model, consumer surplus is $U(q_1, ..., q_{11}) - \sum_{i=1}^{11} q_i p_i$, and welfare is consumer surplus plus the sum of the firms' profits. We assume firm i has constant marginal cost marginal cost c_i and zero fixed costs.

Hospital	Description	Revenue Share in Model
1	HCA	13.90%
2, 3, 4, and 5	HCA acquired	3.20%
6	largest	28.82%
7 and 8	large	17.43%
9, 10, and 11	small	3.20%

Figure 4: Calibrated Revenue Shares

With a parameterized model that mimics the market share characteristics of *Hospital- Corp.*, we can calculate firms' profits, consumers' surplus, and overall social welfare under a variety of scenarios. The scenarios we consider are:

- *Pre-acquisition-noncooperative:*- all eleven firms behave noncooperatively;
- *Post-acquisition-noncooperative:*- firms 1–5 act as a single firm, but that firm and the other six firms behave noncooperatively with respect to one another;
- *Pre-acquisition-cooperative:*- the four largest firms in the pre-acquisition market (firms 1, 6, 7, 8) act as a single firm, but that firm and the other seven firms behave noncooperatively with respect to one another; and
- *Post-acquisition-cooperative:*- firms 1–8 act as a single firm, but that firm and the remaining three firms behave noncooperatively with respect to one another.

For each of these scenarios we can calculate the profit of each firm and the combined profit of firms acting as a single firm. Figure 5 shows how the firms' and various groups of firms' profits change as a result of the acquisitions and as a result of cooperative behavior.

Notice that when firm 1 cooperates with firms 6, 7, and 8 without first making the acquisitions, the combined profits of those four firms increases by only 9%. But if firm 1 first acquires firms 2, 3, 4, and 5, then the cooperative behavior increases the combined profits of firms 1, 6, 7, and 8 by 65%, and it increases the combined profits of firms 1 through 8 by 68% relative to pre-acquisition noncooperative behavior.

One result that is clear from examining these tables is that, given our assumptions and parameterization, when a subset of the firms in an industry collude, the non-colluding firms benefit. For example, in the post-acquisition cooperative case, there

Firm	Post-acquisition noncooperative	Pre-acquisition cooperative	Post-acquisition cooperative
1	12.12%	9.47%	73.50%
2, 3, 4, 5	18.00%	36.38%	84.46%
6	10.67%	8.11%	57.55%
7, 8	13.81%	9.14%	68.55%
9, 10, 11	33.62%	36.38%	331.28%
1+2+3+4+5	14.94%	22.38%	78.76%
1+6+7+8	12.34%	8.82%	65.35%
1+…+8	13.14%	12.72%	68.06%
1+…+11	15.11%	15.00%	93.35%

Figure 5: Change in Proft Relative to Pre-Acquisition Noncooperative

are only three independent firms, firms 9–11, and they experience more than fourfold increases in their profits.[25]

Finally, note that since we assume zero costs, the change in total profit for firms 1–11 is equal to the change in total consumer expenditures, so the last row in Figure 5 shows how consumer expenditures are affected in the different scenarios.

The increases in profits shown in Figure 5 result because the equilibria of the price competition games for the scenarios considered involve higher prices than in the pre-acquisition noncooperative case. Specifically, the increases in prices relative to pre-acquisition noncooperative prices are shown in Figure 6.

Firm	post-acquisition noncooperative	pre-acquisition cooperative	post-acquisition cooperative
1	20.92%	29.41%	116.95%
2, 3, 4, 5	48.45%	16.78%	248.49%
6	5.20%	19.05%	79.84%
7, 8	6.68%	25.78%	103.94%
9, 10, 11	15.59%	16.78%	107.67%

Figure 6: Change in Prices Relative to Pre-Acquisition Noncooperative

[25] Within our model none of our firms is capacity constrained but, in reality, it is possible that capacity constraints limit the gain to the smaller firms. Of course, capacity constraints could be incorporated into the model.

Figure 6 shows that the acquisition itself induces firms 1–5 to increase prices, but by less than 50% relative to the pre-acquisition noncooperative prices. However, the acquisition together with cooperation with firms 6–8 induces firm 1 to more than double its prices and induces firms 2–5 to more than triple their prices, which increase by 248% relative to the pre-acquisition noncooperative prices.

As a result of these price increases, equilibrium quantities change as shown in Figure 7. As shown in Figure 7, the equilibrium quantities of jointly held or co-operating firms are less than those for the pre-acquisition noncooperative case, and the equilibrium quantities of independent firms are larger than for the pre-acquisition noncooperative case, sometimes more than double the pre-acquisition noncooperative quantities.

Firm	post-acquisition noncooperative	pre-acquisition cooperative	Post-acquisition cooperative
1	-7.27%	-15.41%	-20.02%
2, 3, 4, 5	-20.51%	16.78%	-47.07%
6	5.20%	-9.19%	-12.40%
7, 8	6.68%	-13.23%	-17.35%
9, 10, 11	15.59%	16.78%	107.67%
1+2+3+4+5	-15.98%	5.76%	-37.81%
1+6+7+8	3.24%	-12.45%	-16.40%
1+…+8	-3.67%	-3.95%	-25.32%
1+…+11	-0.22%	-0.23%	-1.50%

Figure 7: Change in Quantities Relative to Pre-Acquisition Noncooperative

Given the equilibrium prices and quantities in the various scenarios, we can calculate the change in consumer surplus as a result of the acquisition and subsequent coordination. These calculations show that, although consumer surplus decreases as a result of the acquisition, it decreases by six times as much as a result of the acquisition plus coordination among firms 1–8.[26]

Although the analysis above has focused on a particular cartel in the post-acquisition market, namely the one consisting of firms 1–8, the approach can also provide insights into what cartels we might expect to see in the post-acquisition market. For example,

[26] The effects of the acquisition and subsequent coordination on consumer surplus, and hence welfare, would be larger if the model included capacity constraints that prevented non-coordinating firms from significantly increasing their quantities.

Figure 8 shows that the commonly-owned firms 1–5 benefit from collusion with any of the other firms, but only firms 6, 7, and 8 find the coordination mutually beneficial. The smallest firms, firms 9, 10, and 11, have higher profits if they remain outside the cartel. Similarly, adding firm 7 or 8 to a cartel of 1–6, and adding firm 8 to a cartel of 1–7 generate additional profits for both the original cartel and for the added firm. This suggests that it was appropriate for the FTC to focus on the post-acquisition cartel of firms 1–8, with the three smallest firms remaining out side the cartel.

Base Market Structure	Firm to Add to Cartel	Change in Profit of Original Cartel	Change in Profit of Added Firm
1-5 collude	6	11.14%	7.17%
1-5 collude	7 or 8	10.65%	6.51%
1-5 collude	9, 10, or 11	9.06%	-2.09%
1-6 collude	7 or 8	13.45%	6.56%
1-7 collude	8	20.40%	6.44%
1-8 collude	9, 10, or 11	30.85%	-41.25%

Figure 8: Effects of Incremental Collusion

To conclude this section, we use the above calculations to examine the Herfindahl index in various cases. Figure 9 shows the Herfindahls according to our model of the hospital market in Chattanooga.

Pre-acquisition noncooperative	Post-acquisition noncooperative	Pre-acquisition cooperative	Post-acquisition cooperative
1703	2114	5490	6326

Figure 9: Herfindahl Index

Coate (2005, p.300) states, "the standard Herfindahl index remains appropriate for coordinated interaction cases." In addition, Coate (2005, p.299) states that "a collusion case with a post-merger HHI of 3712 has a 50% chance of a challenge." He continues: "Adding 1000 points to the Herfindahl statistics increases the probability of a challenge to 93%."

As shown in Figure 9, in *Hospital-Corp.*, the post-merger HHI is only 2114 if one assumes the firms behave non-cooperatively, but if one assumes coordination among the top four post-merger firms, the HHI is 6326, well above Coate's range. Thus, an analysis based on HHI's is consistent with the results of our analysis; however, it lacks the ability to quantify the effects of coordination on profits, prices, quantities, and consumer surplus.

3.2.3 Extensions

As an extension to the analysis described above, we can incorporate the potential for post-acquisition improvements in the quality of various hospitals into the analysis.

In our model of the *Hospital-Corp.* acquisitions, the firms are differentiated, with different firms receiving different weights in the representative consumer's utility function. We can view firms that get higher weight in the utility function as offering higher quality. In this sense, in the model described above, firm 1 is medium quality, the firms it acquires (firms 2–5) are poor quality, and firm 6 is high quality.

Consider a claim by firm 1 that, as a result of its acquisition of firms 2–5, the quality of those firms will increase. In general, it might be hard to evaluate and quantify such a claim, but the model offers a way to do this. Specifically, if we just consider the merger and assume no coordinated effects, and if the quality of firms 2–5 increases up to the level of firm 1, then consumer surplus is higher than the pre-acquisition noncooperative level. So, in the absence of coordinated effects, this type of quality improvement would offset the price increases associated with greater concentration. However, one can show that even if the quality of the four acquired firms increases to the level of the high-quality firm, firm 6, consumer surplus still falls as a result of the acquisitions plus coordinated effects (i.e., coordination among firms 1–8).

4 Conclusion

To review, our analytic approach to coordinated effects allows a direct quantification of the incremental payoffs to post-merger collusion among any subset of remaining firms. Any level of collusion can be investigated and specific firms, who might be mavericks, can be isolated. Calibration and estimation can be undertaken with guidance from pre-merger data so that the post-merger simulations are appropriately benchmarked. The analysis may flag specific subsets of firms who may earn extraordinary payoffs from post-merger collusion and, if the merger is approved, these subsets could be monitored for suspicious activities or enjoined ex ante from certain actions as part of merger approval.

With regard to the drawbacks of the approach, because we propose our approach as a supplement to existing analysis, and not a replacement for it, we do not focus on the criticism that it leaves unaddressed issues that are also unaddressed by any existing analysis. It might be argued that our analysis presumes knowledge of the coordinated behavior by firms in the industry who are not participating in the conjectured coordinated behavior. We believe it is unrealistic to think that non-participating firms would view firms engaged in coordinated interaction as acting in full competition. Repeated interaction in the market place will reveal to non-colluding firms that other firms are engaged in coordinated behaviors. Consider, as an example, bidding in procurements by the firms in an industry. If a non-colluding firm observes that another subset of firms is, as a group, bidding less aggressively than in the past, then it is reasonable for the non-colluding firm to infer that there has been a change in rivalry conditions between the subset of firms (all else held constant). Although our analysis is non-dynamic in nature, one should not take that so far as to think that firms do not learn about one another's coordination through their observed actions.

Finally, although not explicitly discussed in the paper, the approach described herein can be used to analyze divestiture and/or entry. The use of the approach to analyze divestiture is illustrated in the *Arch-Coal* example, where Arch Coal is assumed to divest one of Triton's mines to Kiewit. The merger could easily be analyzed with and without the divestiture to quantify the effects of the divestiture. The use of the approach to analyze entry is illustrated in the Appendix, where we consider the effects of fringe coal producers within the context of the *Arch-Coal* example. The role of these fringe producers is similar to that of small entrants into the market. A comparison of the results with and without the fringe producers allows us to quantify the effects of entry.

A Appendix: Extensions of the Auction Example

A.1 Uniform power distributions—fringe

Four bidders plus fringe

Bidder type	Expected surplus
3	2.38
2	1.62
1	0.82
1	0.82
Fringe	3.29
Total surplus	8.94
Expected revenue	82.69

Three bidders plus fringe

Bidder type	Expected surplus
5	4.29
1	0.96
1	0.96
Fringe	3.85
Total surplus	10.07
Expected revenue	81.40
4	3.31
2	1.74
1	0.88
Fringe	3.53
Total surplus	9.47
Expected revenue	82.11
3	2.49
3	2.49
1	0.86
Fringe	3.44
Total surplus	9.28
Expected revenue	82.34
3	2.43
2	1.66
2	1.66
Fringe	3.35
Total surplus	9.10
Expected revenue	82.54

Two bidders plus fringe

Bidder type	Expected surplus
6	5.61
1	1.12
Fringe	4.47
Total surplus	11.20
Expected revenue	80.08
5	4.39
2	1.94
Fringe	3.93
Total surplus	10.26
Expected revenue	81.22
4	3.46
3	2.68
Fringe	3.70
Total surplus	9.84
Expected revenue	81.74

One bidder plus fringe

Bidder type	Expected surplus
7	7.39
Fringe	5.52
Total surplus	12.90
Expected revenue	78.11

Note: Values are distributed over [0,100] according to $F_s(x) = (0.01x)^s$, where s is the bidder type. The fringe is assumed to be four bidders of type 1.

A.2 Uniform power distributions—merger efficiencies

Four bidders		Three bidders		Two bidders	
Bidder type	Expected surplus	Bidder type	Expected surplus	Bidder type	Expected surplus
3	6.16	3	7.02	6	23.83
2	4.34	3	7.02	1	7.25
1	2.24	1	2.58	Total surplus	31.08
1	2.24	Total surplus	16.61	Expected revenue	54.30
Total surplus	14.99	Expected revenue	70.77		
Expected revenue	72.38				
		5	7.72	8	24.40
		3	5.21	1	6.54
		1	1.86	Total surplus	30.94
		Total surplus	14.79	Expected revenue	56.55
		Expected revenue	74.97		

Note: Values are distributed over [0,100] according to $F_s(x) = (0.01x)^s$, where s is the bidder type. The merger is assumed to achieve efficiencies such that a type 2 and a type 1 combine to form a type 5.

A.3 Similar results can be obtained with beta distributions

Four bidders		Three bidders		Two bidders	
Bidder type	Expected surplus	Bidder type	Expected surplus	Bidder type	Expected surplus
High	0.0717	High+Medium	0.1229	High+Medium+Low	0.1911
Medium	0.0317	Low	0.0207	Low	0.0415
Low	0.0127	Low	0.0207	Total surplus	0.2326
Low	0.0127	Total surplus	0.1643	Expected revenue	0.6447
Total surplus	0.1289	Expected revenue	0.7189		
Expected revenue	0.7622				
		High	0.0795		
		Medium+Low	0.0484		
		Low	0.0149		
		Total surplus	0.1428		
		Expected revenue	0.7488		

Note: Values are distributed according to a beta distribution. High, medium, and low types have mean values of 0.8, 0.7, and 0.6, respectively. The standard deviation for all types is 0.2.

A.4(The(numerical(techniques(of(Gayle(and(Richard(2005) allow(any(desired(distributions(to(be(considered

Bidder type	Distribution	Expected surplus
High	Normal	0.14
Medium	Uniform	0.07
Low	Weibull	0.01
Low	Weibull	0.01
Total surplus		0.23
Expected revenue		0.55

Note:%The%high%type%has%values%distributed%according%to%a%normal(0.80,0.25),% truncated to [0,1].%The medium type has values distributed uniformly over [0,1].%Low% types have values distributed according to a Weibull(0.33,1.5), truncated to [0,1].%

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